## CORRESPONDENCE.

EDITOR ANALYST:—

Since you decline to publish my review of Prof. Newcomb's article on Limits on the ground that it is a repetition of arguments already gone over and hence may not be interesting to your readers; I desire to say simply, in regard to the criticism upon myself, that Prof. Newcomb's objection to my definition of a limit is not valid, since, according to accepted definitions including his own, *it is true* that any value of the sine less than unity is the limit of a series of sines subjected to such a law as that there shall be an *indefinite* approach to that value. Also that he has not shown why the syllogism, to which reference was made, is not as applicable to a divided time, as to a divided debt, or to a divided space. It is not, by any means, necessary to assume a case of uniform motion in order to illustrate the *reductio ad absurdum* to which it leads.

DE VOLSON WOOD.

Hoboken, N. J., Aug. 1, 1882.

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SOLUTION OF PROB. 397 BY PROF. J. M. RICE.—Problem 397 will be found in the new edition of Thomson and Tait's Natural Philosophy, page 349. The following is an algebraic solution.

Putting $y' = 0$ we have $x'^2 = x^2 + y^2$, and from the first equation,

$$\varphi(x^2)\,\varphi(y^2) = \varphi(x^2 + y^2)\,\varphi(0) \qquad (a)$$

Again, putting $y^2 = x^2$, $y^2 = 2x^2$, etc., and denoting $\varphi(0)$ by $c$, we have

$$[\varphi(x^2)]^2 = c.\varphi(2x^2), \qquad (b)$$

and $$[\varphi(x^2)]_3 = \varphi(3x^2)c^2, \text{ etc.},$$

finally $$[\varphi(x^2)]^n = \varphi(nx^2)c^{n-1}.$$

We now substitute $z^2$ for $nx^2$ and elminate $n$, whence

$$[\varphi(x^2)]^{1\div x^2} = [\varphi(z^2)^{1\div z^2}C]^{1\div x^2-1\div z^2},$$

or $$\left[\frac{\varphi(x^2)}{c}\right]^{1\div x^2} = \left[\frac{\varphi(z^2)}{c}\right]^{1\div z^2} = k \text{ (a constant)};$$

$$\therefore \varphi(x^2) = ck^{x^2} = ce^{x^2\div h^2}.$$

In Professor Hall's solution of this problem on p. 120, it is assumed that the partial derivatives $df\div du$ and $df\div dv$ are equal [$f$ denoting $f(u, v)$]. I do not see that this assumption is admissible except when $\varphi$ denotes an exponential function.

396. *Selected by Prof. H. T. Eddy.*—"A smooth horizontal disk revol's with the angular velocity $\sqrt{\mu}$ about a vertical axis at which is placed a material particle attracted to a certain point of the disk by a force whose acceleration is $\mu \times$ distance; prove that the path on the disk will be a cycloid. (Routh's Rigid Dynamics, p. 163.)"

Solution by Prof. Asaph Hall.—Let $a$ and $b$ be the coordin's of the attracting point, the origin being at the centre of the disk; and $x$ and $y$ the coordinates of the particle at the time $t$. The attracting force being $[(a-x)^2+(b-y)^2]^{\frac{1}{2}}\times\mu$, the parts of this force resolved along the axes are $(a-x)\mu$ and $(b-y)\mu$. If we consider the axis of $x$ as a radius vector the accelerations along this axis and perpendicular to it are,

$$\frac{d^2x}{dt^2} - x\mu, \qquad \text{and} \qquad 2\sqrt{\mu}\,.\frac{dx}{dt};$$

with similar expressions for the axis of $y$. Hence we have the two equations of motion,

$$\frac{d^2x}{dt^2} - x\mu - 2\sqrt{\mu}\,.\frac{dy}{dt} = (a-x)\mu,$$

$$\frac{d^2y}{dt^2} - y\mu + 2\sqrt{\mu}\,.\frac{dx}{dt} = (b-y)\mu.$$

These give,

$$\frac{d^3x}{dt^3} + 4\mu\,.\frac{dx}{dt} - 2b\mu^{\frac{3}{2}} = 0,$$

$$\frac{d^3y}{dt^3} + 4\mu\,.\frac{dy}{dt} + 2a\mu^{\frac{3}{2}} = 0.$$

If we differentiate these equations in order to remove the constants we shall have two linear differential equations of the fourth order, the solution of which will introduce eight arbitrary constants. Four of these will be determined by the differential equations, and two more by the condition that when $t=0$, $x=y=0$. Putting $2\sqrt{\mu}\,.\,t=\theta$, the solution gives

$$x = \quad c_1 - c_1\cos\theta + c_2\sin\theta + \tfrac{1}{4}b\theta,$$

$$y = -c_2 + c_2\cos\theta + c_1\sin\theta - \tfrac{1}{4}a\theta.$$

These are the equations of a cycloid.

[C. B Seymour, Esq., has also sent a solution of this problem. He finds the equation $-y=\frac{1}{4}\text{ver}\sin^{-1}4x-\sqrt{(\frac{1}{2}x-x^2)}$, and remarks that "This is the equ'n of the path described on the disk by the material particle. It is, as will be seen, a cycloid whose base is the axis of $y$, and whose generating circle has a diameter of one-half; the cycloid lies on the positive side of the axis of ordinates, and for all positive values of $t\sqrt{\mu}$, $y$ is negative."]